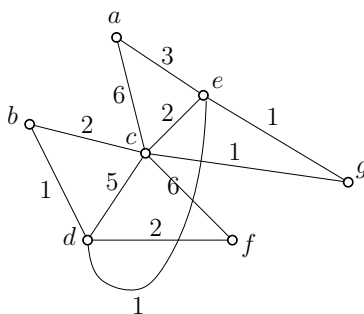


CSCI3160: Special Exercise Set 4

Prepared by Yufei Tao

Problem 1. Let T be a tree. Prove: for any two distinct nodes u, v in the tree, there exists one and exactly one simple path from u to v (a simple path is a path where no vertex appears twice).

Problem 2. Consider the weighted undirected graph below.



Suppose that we run Prim's algorithm to find a minimum spanning tree (MST) of this graph. Explain the order of edges picked by the algorithm.

Problem 3. Consider again the execution of Prim's algorithm in Problem 2. Indicate how the cross edges change as Prim's algorithm runs.

Problem 4 (The Cut-Property) Let $G = (V, E)$ be an undirected connected graph where each edge in E is associated with a positive weight. Consider any non-empty subset $S \subset V$. An edge $\{u, v\}$ in E is an S -cross edge if $u \in S$ but $v \notin S$. Prove: if e is an S -cross edge that has the minimum weight among all S -cross edges, e must belong to some MST of G .