CSCI3160 Design and Analysis of Algorithms (2025 Fall) $${\rm Quiz}\ 3$$

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Please read the following instructions carefully.

- Do not open this quiz booklet until directed to do so.
- Please fill in your Name and Student ID in the corresponding fields above.
- Since the quiz is short, early submissions will not be accepted. Please remain seated until the end of the quiz, even if you finish early.
- This question booklet contains 4 questions across 5 pages, for the total of 100 points. When the quiz begins, please first check to see if any pages are missing.
- This question booklet is printed single-sided. If necessary, feel free to use the blank space on the back of each page for your answers.

This table is intended for grading use only. Exam participants are requested to leave it blank.

Question:	1	2	3	4	Total
Points:	30	25	20	25	100
Score:					

Exam Questions:

1. (30 points) Question statement hidden.

Solution:

- (a) $\{1, 2, 3, 4, 5, 6\}$.
- (b) No. The minimum vertex cover contains 4 vertices only. E.g., $\{2,3,4,6\}$ or $\{2,3,5,6\}$.
- (c) Yes. Optimal solution could be achieved if it picks (2,3) and then (4,6)

2. (25 points) Question statement hidden.

Solution:

Suppose that c(u, v) < 0 and w is any other vertex in the graph. Then, to have the triangle inequality satisfied, we need

$$c(w, u) \le c(w, v) + c(u, v).$$

Now though, subtract c(u, v) from both sides, we get

$$c(w, v) \ge c(w, u) - c(u, v) > c(w, u) + c(u, v)$$

so, it is impossible to have

$$c(w, v) \le c(w, u) + c(u, v)$$

as the triangle inequality would require.

3. (20 points) Question statement hidden.

Solution: $\{6,7,8\},\{3,4,5\}.$

4. (25 points) Question statement hidden.

Solution:

- (a) Assign random values to each variable independently.
- (b) A single-variable, single-clause instance $S = \{(x_1 \lor x_1 \lor x_1 \lor x_1)\}$. The random-assignment algorithm can only achieve an approximation ratio of $\rho = \frac{1}{2}$.