CSCI3350 Introduction to Quantum Computing (2026 Spring)

Recitation

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1. (0 points) Is it always true that

$$(A + B) \otimes C = A \otimes C + B \otimes C$$

for matrices of compatible sizes? Prove your claim.

2. (0 points) Let

$$V = (I \otimes H)\,\mathrm{CZ},$$

where CZ is the controlled-Z gate. Prove that V cannot be written in the form $U_1 \otimes U_2$ for any single-qubit unitaries U_1, U_2.

3. Let $x = (x_0, \ldots, x_{N-1}) \in \{0,1\}^N$, where $N = 2^n$, and let O_x be the standard oracle

$$O_x : |j, b\rangle \mapsto |j,\ b \oplus x_j\rangle .$$

   (a) (0 points) Let y be the string obtained from x by flipping only its last bit x_{N-1}. Using at most one query to O_x, design a circuit that implements one query to O_y.

   (b) (0 points) Let z be the string obtained from x by forcing its last bit to be 1, i.e.,

$$z_{N-1} = 1, \qquad z_j = x_j \text{ for } j \neq N-1.$$

   Using at most one query to O_x, design a circuit that implements one query to O_z.

4. Recall the four Bell states

$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \qquad \left|\Psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

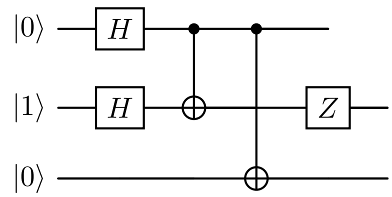
   (a) (0 points) Prove that these four states form an orthonormal basis of $\mathbb{C}^2 \otimes \mathbb{C}^2$.

   (b) (0 points) Give a two-qubit circuit that maps

$$\left|\Phi^{+}\right\rangle \mapsto |00\rangle\,, \quad \left|\Phi^{-}\right\rangle \mapsto |10\rangle\,, \quad \left|\Psi^{+}\right\rangle \mapsto |01\rangle\,, \quad \left|\Psi^{-}\right\rangle \mapsto |11\rangle\,.$$

   Briefly justify your answer.

5. Consider the following quantum circuit:



   (a) (0 points) Compute the final three-qubit state in the computational basis.

   (b) (0 points) If the third qubit is measured in the computational basis at the end, what are the possible outcomes and their probabilities?

6. (0 points) Let

$$|\Psi\rangle_{AB} = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle\,,$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. Suppose Alice applies an arbitrary single-qubit unitary V to her qubit A, while Bob does nothing to qubit B.

Prove that if Bob now measures his qubit in the computational basis, then his outcome probabilities are unchanged by Alice's action. In other words, show that the probabilities of outcomes 0 and 1 remain

$$|a|^2 + |c|^2 \qquad \text{and} \qquad |b|^2 + |d|^2,$$

respectively.

7. (0 points) Let $s \in \{0,1\}^n$ be an unknown bit string. You are given oracle access to the function

$$f_s(x) = s \cdot x \pmod 2,$$

where $s \cdot x$ denotes the bitwise inner product modulo 2. The corresponding oracle acts as

$$O_{f_s} : |x, b\rangle \mapsto |x,\ b \oplus f_s(x)\rangle\,.$$

Design a quantum algorithm that determines s using exactly one query to the oracle. Your answer should include:

- the initial state,
- the sequence of gates,
- and a proof of correctness.

8. Suppose you are given an oracle O_f for a Boolean function $f : \{0,1\}^n \to \{0,1\}$, where

$$O_f : |x, b\rangle \mapsto |x,\ b \oplus f(x)\rangle\,.$$

Let $a \in \{0,1\}^n$ and $c \in \{0,1\}$ be fixed known strings. Define

$$g(x) = f(x \oplus a) \oplus c.$$

   (a) (0 points) Using at most one query to O_f, design a circuit that implements O_g.

   (b) (0 points) Briefly justify why your circuit is correct.